

Fibonacci Numbers

The box below shows a sequence of integers called the **Fibonacci Numbers**. Look carefully at how the sequence has been created.

1,	1,	2,	3,	5,	8,	13,	21,	34,	55,	89	...
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Task A Other Fibonacci Sequences

Copy and continue the following Fibonacci sequences up to the 12th term. They should be created in the same way as the one above.

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|----------------------------|----------------|
| 1) 2, 2, 4, 6, 10, 16..... | 2) 1, 10.... |
| 3) 1, 3, 4, 7..... | 4) 3, 5..... |
| 5) 4, 5, 9, 13..... | 6) 7, 11..... |
| 7) 3, 3, 6... | 8) 1, 100..... |
| 9) 1, 4, | 10) 10, 10.... |

Task B Working Backwards

Here you must work backwards and find the earlier terms in each sequence.

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|----|---|---|---|----|---|----|-----|------|------|---|---|
| 1) | — | — | — | 12 | — | 31 | 50 | 81 | 131 | — | — |
| 2) | — | — | — | — | — | 77 | 125 | 202 | — | — | — |
| 3) | — | — | — | — | — | — | — | 100 | 160 | — | — |
| 4) | — | — | — | — | — | — | — | 1010 | 1750 | — | — |
| 5) | — | — | — | — | — | — | — | 167 | 234 | — | — |

Task C Similar Sequences

The following are not the usual type of Fibonacci sequences. You must work out how they have been created and continue each to the end of the row.

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|----|---|---|---|----|----|-----|----|---|---|----|---|
| 1) | 0 | 1 | 1 | 2 | 4 | 7 | 13 | — | — | — | — |
| 2) | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 6 | 9 | 13 | — |
| 3) | 1 | 2 | 6 | 16 | 44 | 120 | — | — | — | — | — |
| 4) | 2 | 2 | 4 | 8 | 32 | — | — | — | — | — | — |

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Task D Describing Fibonacci Sequences using Algebra

- 1) Continue the following sequences up to the 12th term.
 - a) X, X, ...
 - b) X, Y, ...
 - c) Look again at the sequence in the box at the top of this previous page. What has it got in common with the two algebraic sequences you have just completed?

- 2) The algebraic expression on the right describes how to find the N^{th} term in a Fibonacci sequence using the two previous terms.
$$U_N = U_{N-1} + U_{N-2}$$
 - a) Write down an algebraic expression for finding the:
 - i) 'N+1' th term.
 - ii) The 'N+7' th term.
 - iii) The '2N' th term.
 - b) Write down an algebraic expression for finding the 4th term in a Fibonacci sequence using the 1st and 2nd term.
 - c) Write down an algebraic expression for finding the 10th term in a Fibonacci sequence using the 7th and 8th term.
 - d) Write down an algebraic expression for finding the 12th term in a Fibonacci sequence using the 5th and 7th term.
 - e) Write down an algebraic expression for finding the 20th term in a Fibonacci sequence using the 1st and 2nd term.
 - f) Is it possible to find an N^{th} term for a Fibonacci sequence that uses the first and second terms?

Task E Similar Sequences

Each of the sequences below works in a similar way to Fibonacci sequences. Complete each of them up to the 12th term, then describe them using algebra.

- 1) 0, 1, 1, 2, 4, 7, 13 ...
- 2) 0, 1, 1, 1, 2, 3, 4, 6, 9, 13 ...
- 3) 1, 2, 6, 16, 44, 120 ...
- 4) 1, 2, 2, 4, 8, 32 ...
- 5) 7, 3, 4, -1, 5, -6, 11 ...